Poisson-Lie T-duality: Open Strings and D-branes

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Abstract

Global issues of the Poisson-Lie T-duality are addressed. It is shown that oriented open strings propagating on a group manifold G are dual to D-brane - anti-D-brane pairs propagating on the dual group manifold $\tilde{G}$. The D-branes coincide with the symplectic leaves of the standard Poisson structure induced on the dual group $\tilde{G}$ by the dressing action of the group G. T-duality maps the momentum of the open string into the mutual distance of the D-branes in the pair. The whole picture is then extended to the full modular space $M(D)$ of the Poisson-Lie equivalent σ-models which is the space of all Manin triples of a given Drinfeld double. T-duality rotates the zero modes of pairs of D-branes living on all group targets belonging to $M(D)$. In this more general case the D-branes are preimages of symplectic leaves in certain Poisson homogeneous spaces of their targets and, as such, they are either all even or all odd dimensional.

1 Introduction

This is our first note where we address the global issues of the Poisson-Lie T-duality [1]. We believe that there is a little doubt that Poisson-Lie T-duality does naturally generalize the Abelian [2] –[6] and the traditional non-Abelian [7]–[11] T-dualities. After the original work [1], there was a further development on the subject [12, 13, 14, 15] where there was demonstrated that the Poisson-Lie T-duality enjoys most of the basic characteristic features of the Abelian T-duality at both classical and quantum level. However, in order to complete the full analogy between the standard Abelian and Poisson-Lie T-duality it is crucial to understand the issue of the zero modes or, in other words, the global issues of the Poisson-Lie T-duality. Already in our first paper on the subject dealing with closed strings, we have proved the classical phase space equivalence of the mutually dual sigma-models only for restricted phase spaces deprived of zero modes. Actually we had to restrict the phase space of closed strings of the σ-model on a group manifold G by the constraint of unit monodromy with respect to the dual group and vice versa (for details see [1]). For instance, in the case of the Abelian Drinfeld double (standard Abelian T-duality) the both mutually dual unit monodromy constraints eliminate all momentum and winding zero modes and leave just the oscillator modes. Hence, only the local aspects of T-duality can be recovered in this way. Only in the case of the standard Abelian duality, and remarkably at the quantum level, things become better and for a specific choice of the geometry of the double (the adjustment R and α'/R of the lengths of the dual homology cycles) the duality extends to the zero modes in the standard way [2, 5].

Already for the case of the traditional non-Abelian duality [7, 8, 9, 10, 11] the understanding of the global issues concerning closed strings is generally considered as a difficult problem. A constructive statement was given e.g. in [10] where the authors claimed that the dual CFT is the G orbifold of the original theory and G is the non-Abelian part of the (semi-Abelian) Drinfeld double governing the traditional non-Abelian duality [1]. We are also trying elsewhere to understand the global issues of the Poisson-Lie T-duality for closed strings, however, our hope is to show that Poisson-Lie T-duality is the symmetry of a *single* CFT [16].

Rather surprisingly, we find the global issues of T-duality much easier to treat in the case of open strings. The standard Abelian T-duality for open

strings is now a hot topic. The subject originated in the papers [17, 18, 19, 20] were the appearance of the Dirichlet boundary conditions (hence D-branes) was understood. In this note we shall show that a similar picture emerges in the Poisson-Lie case. The momentum zero modes of open strings are mapped into distance zero modes of pair of D-branes propagating on the dual group manifold and vice-versa. The D-branes propagate as if there were equally big charges with the opposite signs at the ends of the attached Dirichlet strings. These charges feel the field strengths on the D-branes given by the symplectic forms on them. The details we give in section 2.

It may seem that such a T-duality relates objects of different kinds. In fact, we shall show that the T-duality between open strings and pairs of D-branes is just the limiting case of a more general duality between pairs D-branes on G and pairs of D-branes on $\tilde{G}$ group targets. In section 3, we shall describe the construction, which heavily use the results of Dazord and Sondaz [21] and Lu [22] on Poisson structures on G compatible with the action of the Poisson-Lie group G on itself and the results of Drinfeld [23] on Poisson homogeneous spaces of Poisson-Lie groups.

2 Open strings - D-branes duality

Consider a σ-model Lagrangian on a group manifold G

$$L = (R + \Pi(g))^{-1}(\partial_+ g g^{-1}, \partial_- g g^{-1}), \tag{1}$$

where the indices $\pm$ means the light cone variables on the world sheet and R is a nondegenerate bilinear form (with also a non-degenerate symmetric part) on the dual space $\mathcal{G}^*$ of the Lie algebra $\mathcal{G}$ of the group G. $\Pi(g)$ is such a bivector field on the group manifold which gives a Poisson-Lie bracket on G (i.e. the multiplication $G \times G \to G$ is the Poisson map). The model (1) then has a Poisson-Lie symmetry with respect to the right action of G on itself [1][1] and the Poisson bracket $\Pi(g)$ in the standard way furnishes

[1] This Poisson-Lie symmetric model with respect to the right action of G was first constructed in [1]. However, in distinction to the present case, in that original paper we used the *left*-invariant currents instead of the *right*-invariant currents and the σ-model (1) was therefore written in [1] in more cumbersome way. By starting from [15], we use the right-invariant currents and an interested reader may choose [15] as a reference background for reading this note.

the coalgebra $\mathcal{G}^*$ with the Lie algebra structure (for a review see [24]). We denote as $\tilde{\mathcal{G}}$ the coalgebra $\mathcal{G}^*$ with this new Lie algebra structure and $\tilde{G}$ the corresponding group. There is one consistency requirement on the structure constants of the both Lie algebras [24] which is invariant upon the exchanging the algebras. Hence, there is a beautiful duality between the groups G and $\tilde{G}$ discovered by Drinfeld [24]. There is a Poisson-Lie bracket $\tilde{\Pi}(\tilde{g})$ on the dual group manifold $\tilde{G}$ such that it converts the coalgebra $\tilde{\mathcal{G}}^*$ precisely into the original Lie algebra $\mathcal{G}$. In [1] we have argued that the T-duality in string theory is just the manifestation of this Poisson-Lie duality, because for the closed strings deprived of zero modes we have shown that the σ-model (1) is equivalent to the following σ-model on the dual group $\tilde{G}$ manifold:

$$\tilde{L} = (R^{-1} + \tilde{\Pi}(\tilde{g}))^{-1}(\partial_+ \tilde{g}\tilde{g}^{-1}, \partial_- \tilde{g}\tilde{g}^{-1}). \tag{2}$$

The equivalence of the two σ-models at the classical level means the existence of the canonical transformation between the phase spaces of the models which preserves the Hamiltonians.

We have shown in [15] that the phase space of the mutually dual models coincides with the loop group LD of the Drinfeld double D. The Drinfeld double D is a group whose Lie algebra as a vector space is the direct sum of the vector spaces $\mathcal{G}$ and $\tilde{\mathcal{G}}$. The commutators within $\mathcal{G}$ and $\tilde{\mathcal{G}}$ do not change and commutators between $\mathcal{G}$ and $\tilde{\mathcal{G}}$ are given by the combination of the coadjoint actions of $\mathcal{G}$ on $\mathcal{G}^* = \tilde{\mathcal{G}}$ and $\tilde{\mathcal{G}}$ on $\tilde{\mathcal{G}}^* = \mathcal{G}$ [24]. The standard pairing $\langle ., . \rangle$ between the algebra $\mathcal{G}$ and its coalgebra $\tilde{\mathcal{G}}$ is then interpreted as an invariant non-degenerate bilinear form on $\mathcal{D}$ such that $\langle \mathcal{G}, \mathcal{G} \rangle = \langle \tilde{\mathcal{G}}, \tilde{\mathcal{G}} \rangle = 0$ [2]. In [15] we have also written a duality invariant first order (Hamiltonian) action on LD which, upon choosing different parametrizations of LD and solving away 'halves' of fields, yields both σ-models (1) and (2) of the dual pair for the case of the closed strings deprived of zero modes. The explicit form of the Lagrangian of this action is as follows ($l(\sigma, \tau) \in D$)

[2]There exists a different but equivalent way how to construct the bialgebra $(\mathcal{G}, \tilde{\mathcal{G}})$ by starting from a Lie group D (the Drinfeld double) such that its Lie algebra $\mathcal{D}$, viewed as the linear space, can be decomposed into a direct sum of vector spaces which are themselves maximally isotropic subalgebras with respect to a non-degenerate invariant bilinear form on $\mathcal{D}$ [24]. An isotropic subspace of $\mathcal{D}$ is such that the value of the invariant form on any two vectors belonging to the subspace vanishes (maximally isotropic means that this subspace cannot be enlarged while preserving its isotropy). Any such decomposition of the double into a pair of maximally isotropic subalgebras $\mathcal{G} + \tilde{\mathcal{G}} = \mathcal{D}$ is usually referred to as the Manin triple and the pair $(\mathcal{G}, \tilde{\mathcal{G}})$ as the Lie bialgebra.

$$\mathcal{L} = \frac{1}{2}\langle \partial_\sigma l\ l^{-1}, \partial_\tau l\ l^{-1}\rangle + \frac{1}{12} d^{-1}\langle dl\ l^{-1}, [dl\ l^{-1}, dl\ l^{-1}]\rangle + \frac{1}{2}\langle \partial_\sigma l\ l^{-1}, A\partial_\sigma l\ l^{-1}\rangle \tag{3}$$

Here $\langle ., . \rangle$ denotes the non-degenerate invariant bilinear form on the Lie algebra $\mathcal{D}$ of the double. In the second term in the r.h.s. we recognize the two-form potential of the WZW three-form on the double and A is a linear (idempotent) map from the Lie algebra $\mathcal{D}$ of the double into itself. It has two eigenvalues $+1$ and -1, the corresponding eigenspaces $\mathcal{R}_+$ and $\mathcal{R}_-$ have the same dimension $dimG$, they are perpendicular to each other in the sence of the invariant form on the double and they are given by the following recipe:

$$\mathcal{R}_+ = Span\{t + R(t, .), t \in \tilde{\mathcal{G}}\}, \quad \mathcal{R}_- = Span\{t - R(., t), t \in \tilde{\mathcal{G}}\}. \tag{4}$$

Thus the modular space of such actions is described by (non-degenerate) bilinear forms $R(., .)$ (matrices) on the algebra $\tilde{\mathcal{G}}$ [3]. For a better orientation of an interested reader we stress that the first two terms in (3) give together the standard WZW Lagrangian on the double if we interpret τ and σ as the 'light-cone' variables. These two first terms play the role of the 'polarization' term pdq in the first order variational principle

$$S = \int L = \int pdq - Hdt.$$

The remaining third term of the action (3) plays the role of the Hamiltonian H. The field equations coming from (3) are very simple:

$$\langle \partial_\pm l l^{-1}, \mathcal{R}_\mp \rangle = 0. \tag{5}$$

As we have mentioned in the Introduction, the inclusion of the zero modes for the closed strings is a difficult problem. Instead, let us consider an oriented *open* string whose propagation on a group manifold G is governed by the σ-model (1). The boundary conditions at the end-points of the open string are standard: there should be no flow of momentum through the end-points. However, we should be more careful in understanding what the 'momentum' means in this case where the background is not isometric. In our

[3]Since $R(., .)$ is non-degenerate, there exists the inverse bilinear form defined on the dual $\mathcal{G}$ of $\tilde{\mathcal{G}}$, hence such description of the modular space does not break the duality.

case (1) the Noether currents corresponding to the right action of the group G on itself are not conserved on shell (because the dependence on g of the Poisson bracket $\Pi(g)$ spoils the right invariance of the action). The Noether current one-form $J(g)$ is defined by the variation of the action (1)

$$\delta \int L = \int \langle J(g) \stackrel{\wedge}{,} d\epsilon \rangle + \int \epsilon^a \mathcal{L}_{v_a}(L) \tag{6}$$

where $g + \delta g = g(1 + \epsilon)$, $\epsilon \in \mathcal{G}$ and $\mathcal{L}_{v_a}$ are the Lie derivatives of the Lagrangian with respect to the left invariant vector fields on G. Clearly, the (world-sheet) one-form $J(g)$ is an element of the coalgebra $\mathcal{G}^*$ which itself has the Lie algebra structure $\tilde{\mathcal{G}}$. The explicit form of $J(g)$ in terms of the σ-model matrix was given in [1] where it was also shown that for the σ-model (1) the equations of motions can be written as $\tilde{\mathcal{G}}$-valued zero-curvature condition for the $\tilde{\mathcal{G}}$-valued 'connection' form $J(g)$. In other words, a ($\tilde{G}$-valued) quantity

$$\tilde{H} = P \exp \int_\gamma J(g) \tag{7}$$

is conserved. Here P means the path-ordered exponential and γ is an arbitrary curve crossing the world-sheet of the open string. Hence, $\tilde{H}$ is our $\tilde{G}$ valued non-commutative momentum and the boundary conditions are such that the component of the momentum density one-form $J(g)$ along the boundaries vanishes at the end-points of the string. Note that the zero-curvature condition on $J(g)$ means that though $J(g)$ is not a conserved current nevertheless its Wilson line (7) still gives the (non-commutative) conservation law.

In [1] we have argued that every extremal string $g(\sigma, \tau)$ of the model (1) propagating on the group manifold G can be naturally understood as propagating also in the Drinfeld double D. The reason is that for the extremal string the one-form $J(g)$ is the flat $\tilde{\mathcal{G}}$-connection therefore it can be written as

$$J(g) = d\tilde{h}\tilde{h}^{-1}, \quad \tilde{h} \in \tilde{G}. \tag{8}$$

Hence the string propagation on the double can be described by mapping

$$l(\sigma, \tau) = g(\sigma, \tau)\tilde{h}(\sigma, \tau), \tag{9}$$

where g and $\tilde{h}$ are simply multiplied in the Drinfeld double sense. Such $l(\sigma, \tau)$ then fulfils Eqs. (5). Note that given $J(g)$, the corresponding $\tilde{h}$ is given up

to a constant element $\tilde{h}_0 \in \tilde{G}$. This means that the extremal string $g(\sigma, \tau)$ is lifted into the double up to a constant right translation $\tilde{h}_0$ in the double.

What happens to the boundaries of the extremal strings lifted to the double? Because of the boundary conditions, the end-points moverespectively along two copies of the group manifold G, i.e. $G\tilde{h}_i$ and $G\tilde{h}_e$, embedded into the double by the right action of two constant elements $\tilde{h}_i$ and $\tilde{h}_e$ from the dual group $\tilde{G}$. Those elements are constrained by the equation

$$\tilde{h}_e = \tilde{h}_i \tilde{H}, \tag{10}$$

where $\tilde{H}$ is the conserved (non-commutative) momentum of the string. Using our old strategy [1], we may find projections $\tilde{g}(\sigma, \tau)$ into the dual group of the extremal strings $l(\sigma, \tau)$ living in the double according to the relation

$$l(\sigma, \tau) = \tilde{g}(\sigma, \tau) h(\sigma, \tau), \quad h \in G. \tag{11}$$

Under this projection the manifolds $G\tilde{h}_i$ and $G\tilde{h}_e$ of the string end-points in the double get projected by definition into the so-called dressing orbits or orbits of the dressing action of the group G on its dual group $\tilde{G}$ [4] [24]. Those dressing orbits coincide with the symplectic leaves of the Poisson-Lie bracket $\tilde{\Pi}(\tilde{g})$ on $\tilde{G}$ discussed above.

We already know from our previous works [1, 15] that the dynamics of the bulk of the string in the dual group $\tilde{G}$ (corresponding to the σ-model (1) on G) is governed by the σ-model (2) on $\tilde{G}$. Now we have to care about the boundary conditions. We observe that the standard open string boundary conditions on G give rise to the Dirichlet boundary conditions for the open strings propagating on $\tilde{G}$. The end-points of the strings stick on the symplectic leaves of the Poisson-Lie bracket $\tilde{\Pi}(\tilde{g})$ in $\tilde{G}$. These leaves we standartly interpret as the D-branes [19]. They are automatically even dimensional and the mutual geometry of the D-branes corresponding respectively to the string end-points is given by the momentum $\tilde{H}$ of the open string in G. As we have mentioned above by a suitable choice of $\tilde{h}_0$ we are actually free to set the value of $\tilde{h}_i$ to the dual group unit element $\tilde{e}$. Then the submanifold $G\tilde{e}$ of the double coincides with the embedding of the group G into the double D and its projection into $\tilde{G}$ is just one-point symplectic leaf $\tilde{e}$. The whole picture

[4] If the dual group is commutative, then the dressing action of G is just the co-adjoint action of G on its coalgebra $\mathcal{G}^*$.

of duality then becomes particularly transparent: one of the D-branes of the pair is just the origin $\tilde{e}$ of $\tilde{G}$ and the other is the symplectic leaf in $\tilde{G}$ to which belongs the momentum $\tilde{H}$ of the open string in G.

It is obvious, that apart from sticking on the D-branes, something more must hold for the motion of the string end-points *within* the D-branes. Indeed they must move in such a way that their dual open strings in G have vanishing momentum flow through their boundaries. Hence we feel that a boundary term has to be added to the dual model bulk action (2) in order to establish the perfect duality. It is not in fact that difficult to find this boundary term. We may use the first order duality invariant Lagrangian (3) on the double derived in [15] but we have to specify the boundary conditions on the fields $l(\sigma,\tau)$. They are such that in the decomposition $l = g\tilde{h}$ of the double the field g may be arbitrary but $\tilde{h}$ has to be constant along the boundaries of the worldsheet. With such boundary conditions the action coming from the Lagrangian (3) is not well defined because for the opens strings various possible choices of the inverse exterior derivative of the WZW term are inequivalent. We pick up such a choice of d^{-1} of WZW which guarantees that the Polyakov-Wiegmann formula [25] holds[5]

$$\begin{aligned} \int \alpha \ &= \tfrac{1}{2}\textstyle\int \langle \partial_\sigma l\ l^{-1}, \partial_\tau l\ l^{-1}\rangle + \tfrac{1}{12}\textstyle\int d^{-1}\langle dl\ l^{-1}, [dl\ l^{-1}, dl\ l^{-1}]\rangle \\ &= \textstyle\int \langle \partial_\sigma \tilde{h}\tilde{h}^{-1}, g^{-1}\partial_\tau g\rangle. \end{aligned} \tag{12}$$

This WZW action on the double corresponds to the polarization form $\alpha(= pdq)$ part of the first order action (3).

We may now choose the dual parametrization $l = \tilde{g}h$. Then we know that at the end-points of the string, $\tilde{g}$ is constrained to some symplectic leaves. We cannot expect, however, that the Polyakov-Wiegmann formula holds also for the dual parametrization $l = \tilde{g}h$ because another choice of d^{-1} of the WZW form would be needed to ensure this[6]. Fortunately, we have found that those two choices of d^{-1} of WZW term differ [1] by a non-degenerate closed (and hence symplectic) two-form Ω on the double constructed by Semenov-Tian-Shansky [26, 1, 15].

[5]This choice of d^{-1} of WZW form is given by the antisymmetric part of the bilinear form $\langle ., \Pi_{\tilde{L}R} . \rangle$ defined in [1].

[6]The choice of the antisymmetric part of $\langle ., \Pi_{L\tilde{R}} . \rangle$ in the sense of [1].

Hence we may write

$$\int \alpha = \int \langle \partial_\sigma h h^{-1}, \tilde{g}^{-1}\partial_\tau \tilde{g}\rangle + \int \Omega(\tilde{g}h). \tag{13}$$

Because Ω is closed the boundary terms do contribute and the bulk σ-model action (2) gets supplemented with the boundary terms

$$S_b = \int_i d\tau A_\mu(x)\partial_\tau x^\mu - \int_e d\tau A(y^\nu)\partial_\tau y^\nu. \tag{14}$$

Here x^μ and y^ν are some coordinates on the symplectic leaves, $A_\mu(x)$ and $A_\nu(y)$ are electromagnetic potentials on the leaves and the indices i and e denote the end-points of the string. The minus sign between the boundary contributions means that the end-points of the string carry equally large but opposite charges. Obviously, the exact form of the electromagnetic potential come from the Semenov-Tian-Shansky form on the double which induces the standard (coming from the Poisson-Lie structure) symplectic form on the symplectic leaves on $\tilde{G}$. These symplectic forms on the leaves give just the field strenghts of the potentials A_μ.

We have shown that one of the symplectic leaves on $\tilde{G}$, where the end-points of the string live, can be chosen to be the group origin $\tilde{e}$. The opposite end-point then lives on the symplectic leaf which corresponds to the total momentum $\tilde{H}$ of the dual open string living on G and carry a charge which feels the field of the symplectic form on the leaf. In some examples this field is nothing but the field of a monopole sitting at the origin $\tilde{e}$.

3 D-branes - D-branes duality

It may seem that the duality described in the previous section relates apparently different objects: open strings and D-branes. Here we shall show that such a duality is rather a singular case of a more 'symmetrically' looking duality between D-branes and D-branes. Indeed, suppose that our Drinfeld double based on the bialgebra $(\mathcal{G},\tilde{\mathcal{G}})$ has a different decomposition (Manin triple, cf. footnote (6)) into a pair of maximally isotropic subalgebras $\mathcal{K}$ and $\tilde{\mathcal{K}}$. This means that the Drinfeld doubles based on the bialgebras $(\mathcal{G},\tilde{\mathcal{G}})$ and $(\mathcal{K},\tilde{\mathcal{K}})$ respectively coincide. In what follows we shall consider $(\mathcal{K},\tilde{\mathcal{K}})$ and $(\tilde{\mathcal{K}},\mathcal{K})$ as two *different* points of the modular space $M(D)$ of all Manin triples

corresponding to the given Drinfeld double. Whenever we shall speak about a target from $M(D)$, we shall mean a group corresponding to the first Lie algebra of the bialgebra from $M(D)$. Now consider again our 'old' model of open strings on the target G whose dynamics is governed by the σ-model (1). We already now that from the point of view of the dual manifold $\tilde{G}$ this model is equivalent to the σ-model (2) supplemented with the boundary terms (14); how it looks from the point of view of the group manifold K corresponding to $\mathcal{K}$? The answer for the bulk part we already know from [1, 15]; it turns out that the boundary terms also can be elegantly written in terms of the well-known structures on the Drinfeld double. So the action on K reads

$$S = \int d\sigma d\tau (E + \Pi(k))^{-1}(\partial_+ k k^{-1}, \partial_- k k^{-1}) + \int_i A_\mu(x)\partial_\tau x^\mu - \int_e A_\nu(y)\partial_\tau y^\nu, \tag{15}$$

where E is a constant bilinear form on $\tilde{\mathcal{K}}$ related to the form R in (1) by a constant projective transformation [1, 15] and $\Pi(k)$ is the Poisson bracket on K corresponding to $\tilde{\mathcal{K}}$. The meaning of A's will become clear soon and the coordinates x and y parametrize the projections into K of the submanifolds $G\tilde{h}_i$ and $G\tilde{h}_e$ of the double. Recall that those are submanifolds where the end-points of the string live. The projection $k(\sigma,\tau)$ of the string from the double into K is defined in the standard way:

$$l(\sigma,\tau) = k(\sigma,\tau)\tilde{m}(\sigma,\tau), \quad \tilde{m} \in \tilde{K}. \tag{16}$$

Now we use the result of Dazord and Sondaz [21] and Lu [22] which states that all Poisson (but not necessarily Poisson-Lie!) structures on the group K compatible with the right action of the Poisson-Lie group K on itself[7] are in one-to -one correspondence with the isotropic subalgebras in $\mathcal{D}$ transversal to $\mathcal{K}$ (=having vanishing intersection with $\mathcal{K}$). Suppose that from the point of view of $\mathcal{K}$ the algebra $\mathcal{G}$ is indeed such a transversal algebra. Using the results of [21, 22] we can easily find the symplectic leaves of this Poisson-structure on K. One has first to exponentiate the transversal isotropic subalgebra (in

[7]Compatibility means that the multiplication map $K \times K \to K$ is a Poisson map. Of course, here the first copy of K and the image K has the Poisson structure in question and the second copy of K which acts from the right has the Poisson-Lie structure induced by the dual group $\tilde{K}$.

our case $\mathcal{G}$) into the corresponding subgroup of the double (G), then shift it by the right action of an arbitrary constant element r from the double D and finally project the submanifold of D obtained in this way into K. Remarkably, it follows that the end-points of our open string in K move just along these symplectic leaves of the Poisson-structure induced on K by the isotropic algebra $\mathcal{G}$. Moreover, the potential $A_\mu(x)$ has the field strength equal to the symplectic form induced on the leaf by the Poisson structure on K induced by $\mathcal{G}$. The mutual geometry of the symplectic leaves, or D-branes, in K is given by the momentum $\tilde{H}$ of the corresponding open string in the target G.

If the algebra $\mathcal{G}$ is not transversal to $\mathcal{K}$ then we have to use a generalization of the results [21, 22] given by Drinfeld [23]. The D-brane structure on K is, of course, also obtained in this case by projecting the submanifolds $G\tilde{h}_i$ and $G\tilde{h}_e$ into K. But now the D-branes need not be symplectic leaves of a Poisson structure on K. It turns out, however, that they *are* preimages of the symplectic leaves of certain Poisson komogeneous K-space[8]. This Poisson homogeneous K-space is obtained as a left coset of K by a subgroup whose Lie algebra is the intersection of $\mathcal{G}$ and $\mathcal{K}$. The Poisson structure on it is induced by $\mathcal{G}$ in the sense of [23]. The pullback into K of the symplectic form on a symplectic leaf in this coset K-space gives the field stregth on the pre-image of the leaf(=D-brane) in K. In particular, if $\mathcal{G}$ is $\mathcal{K}$ itself then the coset is just one point, its pre-image and hence D-brane is the whole group G and the field strength on the D-brane trivially vanishes. This is the D-brane interpretation of the open strings in a target G. Note that the dimensions of D-branes for an arbitrary target K from the modular space $M(D)$ are either all even or all odd dimensional. This follows from the fact that the dimension of the D-brane is equal to the dimension of the corresponding symplectic leaf in the coset plus the dimension of the intersection of $\mathcal{G}$ and $\mathcal{K}$.

Thus we may say that, in general, the duality we are describing just rotates D-branes on various targets from $M(D)$. It is clear that the structure of the boundary terms which supplement the bulk actions of the σ-model from $M(D)$ (cf. Eq. (15)) is given solely by an isotropic subalgebra of the double. There is an interesting possibility that this isotropic subalgebra $\mathcal{F}$

[8]A Poisson homogeneous K-space is a homogeneous K-space equipped with a Poisson structure compatible with the action of the Poisson-Lie group K. Drinfeld has shown in [23] that all such Poisson homogeneous spaces correspond to maximally isotropic subalgebras of the double.

may not belong to $M(D)$ which means that it does not possess an isotropic dual subalgebra $\tilde{\mathcal{F}}$. Then our duality would rotate just 'pure' D-branes in all targets from $M(D)$ which means that there is no target in $M(D)$ for which the corresponding D-branes would be equivalent just to open strings. We may therefore conclude by stating the main result of our note:

Theorem: A Poisson-Lie equivalent class of D-brane theories is given by a pair: a Drinfeld double D and a maximally isotropic subalgebra $\mathcal{F}$ in it. For an arbitrary group target K from $M(D)$ the D-brane theory dual to the other theories in $M(D)$ is described by the action of the form (15) with the appropriate form E [9]. The D-branes coincide with the preimages of the symplectic leaves of the Poisson structure in the left coset of the group K by the subgroup given by the intersection of $\mathcal{K}$ and $\mathcal{F}$. The field strength of the boundary potential A_μ on the D-brane is given by the pullback of the symplectic form on the symplectic leaf. The mutual geometry of the D-branes in the pair is given by one constant element d_0 from the double (more precisely by an element of the left coset of the double by the group F having the Lie algebra $\mathcal{F}$). This element measures the 'distance' of the two copies of the group manifold F lifted in the double by right action of two arbitrary constant elements from the double.

The proof of the Theorem is simple and more or less straightforward.

Remark: In the case of the Abelian T-duality the Drinfeld double is a $2d$-dimensional Abelian group and the modular space $M(D)$ is the coset $O(d, d.R)/O(d, R) \times O(d, R)$ (for the simply connected double). An arbitrary maximally isotropic subalgebra $\mathcal{F}$ in the double does have its dual $\tilde{\mathcal{F}}$ and, hence, it is an element of $M(D)$. All corresponding D-branes theories are then equivalent to the open string theory on the target F.

4 Outlook

Among easy open problems which should be addressed we may mention an understanding of Buscher's duality for D-branes where the σ-model manifold is a product of some manifold M and a group G and the σ-model is Poisson-

[9] $E(.,.)$ is given by writing the subspaces $\mathcal{R}_\pm$ in the same way as in (4) but now with $t \in \tilde{\mathcal{K}}$.

Lie symmetric with respect to the right action of the group G on the target. Another not difficult problem would be a path integral argument which would provide a quantization of the described global picture of D-branes duality. We shall probably solve these problems in near future.

It appears more difficult (but not less important) to provide a supersymmetric generalization of the formalism since, to our knowledge, a concept of a super-double is not yet developed. Because the pair of a double and of its maximally isotropic subalgebra is nothing but a classical limit of the quasi-Hopf algebras [27] there appears a tantalizing possibility of a relevance of the quasi-Hopf algebras in a CFT description of the D-branes Poisson-Lie T-duality. It seems very probable that the duality of the quantum D-branes is governed by the geometry and the representation theory of quantum groups. We believe that many of the results obtained in past in the field of quantum groups will find a direct application in string theory in this way.

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References

[1] C. Klimčík and P. Ševera, Phys. Lett. B351 (1995) 455, hep-th/9502122; P. Ševera, *Minimálne Plochy a Dualita*, Diploma thesis, Prague University, May 1995, in Slovak; C. Klimčík, *Poisson-Lie T-duality*, Lecture given at Trieste Conference on S-duality and Mirror Symmetry, June 1995, to appear in Nucl. Phys. Proc. Suppl., hep-th/9509095

[2] K. Kikkawa and M. Yamasaki, Phys. Lett. B149 (1984) 357; N. Sakai and I. Senda, Prog. Theor. Phys. 75 (1986) 692

[3] T.H. Buscher, Phys. Lett. B194 (1987) 51, B201 (1988) 466

[4] A. Giveon, E. Rabinovici and G. Veneziano, Nucl. Phys. B322 (1989) 167; K.A. Meissner and G. Veneziano, Phys. Lett. B267 (1991) 33; A.A. Tseytlin, Nucl. Phys. B350 (1991) 395

[5] M. Roček and E. Verlinde, Nucl. Phys. B373 (1992) 630; A.A. Tseytlin, Mod. Phys. Lett. A6 (1991) 1721; A.S. Schwarz and A.A. Tseytlin, Nucl.

Phys. B399 (1993) 691; A. Giveon and M. Roček, Nucl. Phys. B380 (1992) 128; E. Kiritsis, Mod. Phys. Lett. A6 (1991) 2871; Nucl. Phys. B405 (1993) 109

[6] E. Álvarez, L. Álvarez-Gaumé, J. Barbón and Y. Lozano, Nucl. Phys. B415 (1994) 71; A. Giveon, M. Porrati and E. Rabinovici, Phys. Rept. 244 (1994) 77; E. Álvarez, L. Álvarez-Gaumé and Y. Lozano, Nucl. Phys. Proc. Suppl. 41 (1995) 1; Phys. Lett. B336 (1994) 183; C. Klimčík and P. Ševera, Mod. Phys. Lett. A10 (1995) 323, hep-th/9411003

[7] B.E. Fridling and A. Jevicki, Phys. Lett. B134 (1984) 70

[8] E.S. Fradkin and A.A. Tseytlin, Ann. Phys. 162 (1985) 31

[9] X. de la Ossa and F. Quevedo, Nucl. Phys. B403 (1993) 377

[10] A. Giveon and M. Roček, Nucl. Phys. B421 (1994) 173

[11] M. Gasperini, R. Ricci and G. Veneziano, Phys. Lett. B319 (1993) 438; E.Álvarez, L. Álvarez-Gaumé and Y. Lozano, Nucl. Phys. B424 (1994) 155; S. Elitzur, A. Giveon, E. Rabinovici, A. Schwimmer and G. Veneziano, Nucl. Phys. B435 (1995) 147; E. Kiritsis and N. Obers, Phys. Lett. B334 (1994) 67; K. Sfetsos, Phys. Rev. D50 (1994) 2784; T. Curtright and C. Zachos, Phys. Rev. D49 (1994) 5408; O. Alvarez and C-H. Liu, *Target-Space Duality between Simple Compact Lie Groups and Lie Algebras under the Hamiltonian Formalism: I. Remnants of Duality at the Classical Level*, preprint UMTG-184, hep-th/9503226; Y. Lozano, Phys.Lett. B355 (1995) 165

[12] E. Tyurin, *Non-Abelian Axial-Vector duality: a Geometric Description*, hep-th/9507014

[13] A.Yu. Alekseev, C. Klimčík and A.A. Tseytlin, *Quantum Poisson-Lie T-duality and WZNW model*, preprint CERN-TH/95-251, hep-th/9509123

[14] E. Tyurin and R. von Unge, *Poisson-Lie T-duality: the Path-Integral Derivation*, hep-th/9512025

[15] C. Klimčík and P. Ševera, *Poisson-Lie T-duality and Loop Groups of Drinfeld Doubles*, preprint CERN-TH/95-330, hep-th/9512040

[16] C. Klimčík and P. Ševera, *in preparation*

[17] P. Hořava, Phys. Lett. B231 (1989) 251; Nucl. Phys. B327 (1989) 461; A. Sagnotti, *Open Strings and Their Symmetry Groups*, in Cargése 87, *Non-perturbative Quantum Field Theory*, ed. G. Mack et al., (Pergamon Press 1988) p.512; M. Bianchi and A. Sagnotti, Phys. Lett. B231 (1989) 389

[18] M. Green, Phys. Lett. B201 (1988) 42 and B266 (1991) 325

[19] J. Polchinski, *Dirichlet Branes and Ramond-Ramond Charges*, hep-th/9510017

[20] J. Polchinski and E. Witten, *Evidence for Heterotic-Type I String Duality*, hep-th/9510169

[21] P. Dazord and D. Sondaz, *Groupes de Poisson Affines*, Séminaire Sud-Rhodanien de Géométrie 1989, P. Dazord et A. Weinstein éd, MSRI publications, Springer-Verlag, 1990

[22] J.H. Lu, *Multiplicative and Affine Poisson Structures on Lie Groups*, Ph.D. thesis, University of California, Berkeley, 1990

[23] V.G. Drinfeld, Teoreticheskaya i Matematicheskaya Fizika 95 (1993) 226

[24] V.G. Drinfeld, *Quantum Groups*, in Proc. ICM, MSRI, Berkeley, 1986, p. 708; F. Falceto and K. Gawędzki, J. Geom. Phys. 11 (1993) 251; A.Yu. Alekseev and A.Z. Malkin, Commun. Math. Phys. 162 (1994) 147

[25] A. Polyakov and P. Wiegman, Phys. Lett. B131 (1983) 121

[26] M.A. Semenov-Tian-Shansky, Publ. RIMS, Kyoto University 21 (1985) 1237

[27] V.G. Drinfeld, Leningrad Math. Journal 1 (1990) 1419